# Chapter 7
# Risk-Management Models Based on the Portfolio Theory Using Historical Data under Uncertainty

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## ABSTRACT

*This chapter considers various types of risk-management models based on the portfolio theory under some social uncertainty that received historical data includes ambiguity, and that they are assumed not to be constant. These models with uncertainty are represented many social problems such as assets allocation, logistics, scheduling, urban project problems, etc.. However, since these problems with uncertainty are formulated as stochastic and fuzzy programming problems, it is difficult to solve them analytically in the sense of deterministic mathematical programming. Therefore, introducing possibility and necessity measures based on the fuzzy programming approach and considering the concept of risk-management based on the portfolio theory, main problems are transformed into the deterministic programming problems. Then, in order to solve the deterministic problems efficiently, the solution method is constructed.*

## INTRODUCTION

A mission in this chapter is to extend previous mathematical programming problems using historical data to stochastic and fuzzy programming problems under uncertainty, particularly considering the risk-management. In the practice decision making, it is necessary to take various constraints and assumptions into consideration as well as uncertainty, such as the probability derived from the statistical analysis of historical data and the ambiguity derived from lack of reliable information and decision maker's subjectivity. Since it is difficult that decision makers know precise information due to such uncertainty, they need to make appropriate decisions under uncertainty.

Until now, in order to deal with uncertainty in the sense of mathematical programming, many studies with respect to stochastic and fuzzy programming have been performed. Stochastic programming is

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a field of mathematical methods to deal with the optimization problems under uncertainty characterized by stochastic fluctuation. The application areas of stochastic programming include many fields (inventory, finance and marketing, etc.). Particularly, the portfolio selection problem, which is combined probability and optimization theory with the investment behavior, is one of the most important problems in stochastic programming problems. Since portfolio theories have focused on risk management under random and ambiguous conditions, they have greatly advanced since the initial important study proposed by Markowitz. Then, some measures for the risk management have been developed such as Semi-variance model, Absolute deviation model, Safety first model, Value-at-Risk, etc.. Thus, in the research field of portfolio selection problem, researchers have studied models with uncertainty and proposed efficient and versatile models of appropriate risk management. Furthermore, some researchers have also considered applying the portfolio theory to general mathematical programming problems such as the asset allocation in production processes and logistics. Therefore, the stochastic programming based on the portfolio theory has become an important field to the mathematical programming from the view point of theory as well as practice.

On the other hand, with respect to the ambiguity such as lack of reliable information and decision maker's subjectivity, they are assumed to be rather fuzziness than randomness. The centre of social behavior in economy, investment and production fields is the human behavior, and so it is obvious that psychological aspects of decision makers have a major impact on social behaviors. Then, it is also clear that some factors in historical data include the ambiguity. Therefore, in order to represent such ambiguity and subjectivity, a fuzzy number was introduced in some previous researches. The fuzzy number is roughly a number to represent the degrees of attribution and preference to objectives directly. Thus, the concept of fuzzy number is different from that of random variable. Many previous researches have been dealt with random variables or fuzzy numbers in mathematical programming problems, separately. However, practical social systems obviously weave such randomness with fuzziness. Therefore, in the case that decision makers consider the present social problems as mathematical programming problems, they need to consider not only randomness but also fuzziness, simultaneously.

Therefore, by extending risk management models using historical data based on the portfolio theory, standard stochastic and fuzzy programming models under uncertainty to general decision making problems, we propose several types of general risk-management problems under uncertainty. The proposed models include many previous stochastic and fuzzy programming problems by considering the parameters setting, and so they become very versatile problems to apply various social situations. However, in the sense of the deterministic mathematical programming, these mathematical models are not well-defined problems due to random and fuzzy variables, and so we need to set a criterion for each objective or constraint involving random and fuzzy variables in order to solve them analytically using the mathematical programming. In this chapter, using the standard stochastic and fuzzy programming approaches, we perform the deterministic equivalent transformations to main problems. Furthermore, deterministic equivalent problems derived from such stochastic and fuzzy programming problems are generally complicate problems, and so it is often difficult to apply standard programming approaches to these problems. Therefore, we develop the efficient and analytical solution method to each proposed model considering the analytical strictness and simple usage in practical social situations.

This chapter is organized as follows. In Background section, we provide the brief literature review of stochastic, fuzzy programming and portfolio selection problems under uncertainty. Then, we introduce some risk-management models