Chapter 6
Image Theory for Electrical Impedance Tomography

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ABSTRACT

Image reconstruction by electrical impedance tomography is, generally, an ill-posed nonlinear inverse problem. Regularization methods are widely used to ensure a stable solution. Herein, we present a novel electrical impedance tomography algorithm for reconstruction of layered biological tissues with piecewise continuous plane-stratified profiles. The algorithm is based on the reconstruction scheme for piecewise constant conductivity profiles, which utilizes Legendre expansion in conjunction with improved Prony method. This reconstruction procedure, which calculates both the locations and the conductivities, repetitively provides inhomogeneous depth discretization, (i.e., the depths grid is not equispaced). Incorporation of this specific inhomogeneous grid in the widely used mean least square reconstruction procedure results in a stable and accurate reconstruction, whereas, the commonly selected equispaced depth grid leads to unstable reconstruction. This observation establishes the main result of our investigation, highlighting the impact of physical phenomenon (image theory) on electrical impedance tomography, leading to a physically motivated stabilization of the inverse problem, (i.e., an inhomogeneous depth discretization renders an inherent regularization of the mean least square algorithm).

INTRODUCTION

The first step of medical treatment is the diagnosis. This investigation is focused on some fundamental mathematical and physical characteristics of such diagnostic tool, namely the Electrical Impedance Tomography (**EIT**), a subject of great scientific and public interest (Holder, 2005; Brown, 2003; Lionheart, 2004). The basic EIT's assumption is that different biological tissues can be distinguished by their conductivities. This knowledge allows one to acquire medical insight on the biological structure by **reconstruction** of its electrical characteristics. Many reconstruction techniques

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were developed during the last two decades. One of the most recently emerged techniques is the EIT. In this approach the, generally complex, conductivity profile of the tissues under reconstruction, is estimated by processing the quasistatic electromagnetic field data, and measured on the surrounding boundary.

EIT method is currently investigated for variable possible medical imaging and detection applications. For example, it appears appropriate for non-invasive cardiac stroke volume measurements, as the thoracic conductivity distribution is altered during the cardiac cycle (Zlochiver, 2006). Additional medical implementations include early breast and skin cancer detection (Assenheimer, 2001), biofilms thickness monitoring (Linderholm, 2007), and excitation with wireless (induced-current) electrodes (Zlochiver, 2003; Levy, 2002).

In spite of its advantages as a noninvasive, inexpensive, and potentially highly informative medical imaging modality, several factors limit the performance of EIT and prevent its adoptation as a clinically viable technique. These limitations can be classified into two groups: "physical" and "mathemtical". While the physical limitations may be addressed by improvements in instrumentation, the mathematical limitations are fundamental, and attempts to miligate them in the **reconstruction** procedure involve various, often undesirable performance compramises (Levy, 2002).

Herein, we are motivated to move one step forward in the process of incorporating analytic derivations into the solution of inverse problems. Specifically, we use the analytic solution of the forward problem associated with **plane stratified***n* layered media (Livshitz, 2001; Einziger, 2002; Einziger, 2005), for the solution of the EIT inverse problem. As it will be shown throughout the study, the pitfalls to be overcome in EIT **reconstruction** procedures rise in the presented media as well. However, the advantage of analytic solution, derived for this media, is exploited and enable to investigate the origin of the characteristic common failures of the existing EIT algorithms.

In applying EIT techniques (Dolgin, 2004; Dolgin, 2006) two crucially important issues have to be clarified and specified: (i) an explicit relation connecting the inverse procedure and the forward problem; (ii) a locality principle, linking between the electrical impedance spacial distribution and the corresponding quasistatic data, measured on the surrounding boundary. The first issue is resolved herein by utilizing a recently proposed image series expansion scheme for layered media, resulting in a novel reconstruction method. Furthermore, in the WKB limit leading to a one to one mapping between each image term and a corresponding layer the second issue is also resolved. For non-WKB image series expansion, however, where each image term corresponds to a specific layer and its neighborhood the novel **EIT** procedure can be readily modified as to account for a partial loss of the locality.

This work links between image theory (Einziger, 2002; Einziger, 2005) and an EIT model for reconstruction of **plane stratified** media. We begin with the **reconstruction** of piecewise constant layered media by a novel two step efficient algorithm utilizing Legendre transform and improved Prony Method (PM) (Dolgin, 2004). Computations based on this procedure, including noisy data, provide accurate, efficient and stable reconstruction, particularly, when relatively small data base and, consequently, shallow penetration are required. Furthermore, our methods holds promising for an efficient reconstruction of locally stratified media (relative to the Legendre window width). Then, the reconstruction procedure is extended as to incorporate piecewise continuous layered media. Each continuous layer is discretized and then reconstructed as a collection of a piecewise constant layers set. The algorithm provides accurate reconstruction results for moderate depths. This reconstruction procedure, which calculates both the locations