

## Chapter 2

# Hydrodynamic Equations from Kinetic Theory: Fundamental Considerations

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### ABSTRACT

*In this chapter, a theoretical description is provided for the solid (granular) phase of the gas-solidss flows that are the focus of this book. Emphasis is placed on the fundamental concepts involved in deriving a macroscopic hydrodynamic description for the granular material in terms of the hydrodynamic fields (species densities, flow velocity, and the granular temperature) from a prescribed “microscopic” interaction among the grains. To this end, the role of the interstitial gas phase, body forces such as gravity, and other coupling to the environment are suppressed and retained only via a possible non-conservative external force and implicit boundary conditions. The general notion of a kinetic equation is introduced to obtain macroscopic balance equations for the fields. Constitutive equations for the fluxes in these balance equations are obtained from special “normal” solutions to the kinetic equation, resulting in a closed set of hydrodynamic equations. This general constructive procedure is illustrated for the Boltzmann-Enskog kinetic equation describing a system of smooth, inelastic hard spheres. For weakly inhomogeneous fluid states the granular Navier-Stokes hydrodynamic equations are obtained, including exact integral equations for the transport coefficients. A method to obtain practical solutions to these integral equations is described. Finally, a brief discussion is given for hydrodynamics beyond the Navier-Stokes limitations.*

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## INTRODUCTION

Activated granular materials occur ubiquitously in nature and practical realizations in industry. Many of the phenomena occur on length scales that are large compared to the size of constituent particles (grains) and time scales long compared to the time between collisions among the grains. In this case a description of the system in terms of the values for hydrodynamic fields in cells containing many particles, analogous to molecular fluids, can apply for granular fluids. The hydrodynamic fields for molecular fluids are the densities associated with the globally conserved quantities. In the absence of reactions, these are the species densities, total momentum density, and the energy density. More commonly the momentum density is replaced by a corresponding flow field, and the energy density is replaced by a related temperature. The time dependence of such a macroscopic description (hydrodynamics) follows from the exact conservation equations for these fields, supplemented by “constitutive equations” providing a closed description in terms of the fields alone.

The key difference between granular and molecular fluids is that the former involves collisions between macroscopic grains. These collisions conserve momentum but dissipate energy since part of the kinetic energy of the grains goes into micro-deformations of the surface and exciting other internal modes of the grains. Even so, a hydrodynamic description for a fluid of grains can be given under appropriate conditions, following closely the approach developed in the context of molecular fluids, starting from the exact “balance equations” for the densities of interest. The objective of this chapter is to provide an overview of how general constitutive equations can be obtained from a fundamental basis in kinetic theory. The discussion does not make specific reference to a particular fluid state or kinetic theory. This overview is followed by a practical illustration for the special case of Navier-Stokes hydrodynamics for weakly non-uniform states, derived from the generalized Enskog kinetic theory (van Beijeren & Ernst 1973, 1979) extended to granular systems (Brey, Dufty & Santos 1997; see also Appendix A of Garzo, Dufty & Hrenya 2007). Extensive references to previous work on Navier-Stokes constitutive equations from Boltzmann and Enskog kinetic theories can be found in the review of Goldhirsch 2003, the text of Brilliantov & Poschel 2004, and in the recent articles Garzo, Dufty, & Hrenya 2007 and Garzo, Hrenya & Dufty 2007.

The balance equations are local identities expressing the change in hydrodynamic fields of a cell due to their fluxes through the boundaries of that cell and local sources within the cell. The central problem is to represent the fluxes and sources in terms of these hydrodynamic fields and their gradients. In many cases the form of these constitutive equations is known from experiments (e.g., Fick’s diffusion law, Newton’s viscosity law). An important advantage of kinetic theory as the basis for constitutive equations, in contrast to such phenomenology generalized from experiment, is that both quantitative and qualitative predictions follow as mathematical consequences of the theory. Thus the form of the hydrodynamic equations, the values of their parameters, and the validity conditions for applications are provided as one unit. In practice, most applications to granular fluids have focused on low density conditions and moderate densities at low dissipation (e.g., the Boltzmann and Enskog kinetic theories) (Jenkins & Mancini 1989, Jenkins 1998, Lun 1991). However, the approach emphasized here is more general and provides a means to describe quite general complex fluid states such as those that occur more generally for granular fluids. The aim of this chapter is to provide a pedagogical overview of the basis for hydrodynamics as arising from kinetic theories (for a similar analysis based on the low density Boltzmann equation see Dufty & Brey 2005). With this goal in mind, attention is restricted to the simplest case of smooth grains interacting through pair-wise additive short ranged interactions. Other important effects

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