Chapter 10
Quantum Phase–Hebbian Image Processing

In this chapter, some specific characteristics of quantum-implementable phase-Hebbian content-addressable associative memory and pattern recognition are discussed. Quantum formalism, constrained by the closure relation, is generalized into a flexible information-processing system by a suitable (re)interpretation of quantum states involving "fuzzification" of the orthonormality and closure relations.

Core of the quantum associative net. Let us first repeat the core of the Quantum Associative Network model. In Feynman's path-integral formalism, the Schrodinger equation can be rewritten (AuxLit 16) in the form

$$\Psi(\vec{r}_2, t_2) = \int\int G(\vec{r}_1, t_1, \vec{r}_2, t_2)\Psi(\vec{r}_1, t_1)d\vec{r}_1 dt_1 \tag{10.1}$$

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where the kernel G is the propagator or the Green function having form of a projection-operator (Bjorken & Drell, 1964/65):

$$G(\vec{r}, t_1, \vec{r}_2, t_2) = \sum_{k=1}^{P} \psi_k(\vec{r}_1, t_1)^* \psi_k(\vec{r}_2, t_2) \tag{10.2}$$

The wave-function Ψ is a superposition of P eigen-wave-functions ψ_k which can represent plane-waves

$$\psi_k(\vec{r}, t) = A_k(\vec{r}, t) e^{i\varphi_k(\vec{r}, t)} \tag{10.3}$$

or wavelets (Lee, 1996; Schempp, 1994, 1995).[1] For simplicity we will not use the Gabor wavelets as wave-packets ψ_k in the formalism explicitly, but this neuropsychologically-important option is allowed.

It is assumed that it is possible to encode information into quantum eigenwaves ψ_k: we let an *eigen-wave-function represent an image*. For each possible vector-basis ψ_k (k = 1,..., P) there is an expression of the same type as Equation (10.2) (Messiah, 1965) which "stores" the eigenpatterns and performs projections to eigen-subspaces. Now it will be shown how such a quantum system can be manipulated in order to realize content-addressable memory storage and associative retrieval.

Opening the closure relation. Because propagator (10.2) must reproduce the initial state in dynamical Equation (10.1) when $t_1 = t_2$, the quantum closure relation

$$\sum_{k=1}^{P} \psi_k(\vec{r}_1, t)^* \psi_k(\vec{r}_2, t) = \delta(\vec{r}_1 - \vec{r}_2) \text{ or } \sum_{k=1}^{P} \psi_k(\vec{r}_1)^* \psi_k(\vec{r}_2) = \delta(\vec{r}_1 - \vec{r}_2) \tag{10.4}$$

must be satisfied (Messiah, 1965).

Closure relation (10.4) implies the postulate of complete and orthonormal set of quantum eigenstates: i.e $\Psi = \sum_k c_k \psi_k$ (completeness), and the scalar product of eigenvectors ψ_k,which have norm 1, is 0 (orthonormality).

Prescription (10.4) ensures reversible and unitary quantum evolution[2] determined by the linear Schrodinger equation (implying complete orthonormal set of eigenwaves in kernel— Equation (10.2)) *if* the system is closed, i.e. if there is no disturbance from environment. On the other hand, the same kernel, (10.2), serves as a projection-operator realizing non-unitary, non-linear and irreversible "collapse of the wave-function" *if* the system is *open*, i.e. if there is a disturbance from environment (Wheeler & Zurek, 1983). If we "perturb" the system, incorporating an "informa-