Chapter 3.11
Semi–*E*–Preinvex Functions

Yu-Ru Syau
National Formosa University, Taiwan

E. Stanley Lee
Kansas State University, USA

ABSTRACT

A class of functions called semi-E-preinvex functions is defined as a generalization of semi-E-convex functions. Similarly, the concept of semi-E-quasiconvex functions is also generalized to semi-E-prequasiinvex functions. Properties of these proposed classes are studied, and sufficient conditions for a nonempty subset of the n-dimensional Euclidean space to be an E-convex or E-invex set are given. The relationship between semi-E-preinvex and E-preinvex functions are discussed along with results for the corresponding nonlinear programming problems.

1. INTRODUCTION

Convexity plays an important role in optimization theory. Various generalizations of convex functions have appeared in the literature. A significant generalization of convex functions is the introduction of preinvex functions, originated by (Hanson and Mond, 1987) but so named by (Jeyakumar, 1985). Later, (Weir and Mond, 1988) studied how and where preinvex functions can replace convex functions in multiple objective optimization.

Recently, (Youness, 1999) introduced the concepts of E-convex sets and E-convex functions

which generalize the definitions of convex sets and convex functions based on the effecting of an operator E on the domain on which functions are defined. Youness's initial results inspired a great deal of subsequent work, see (Duca et al., 2000; Duca & Lupsa, 2006; Fulga & Preda, 2009; Syau & Lee, 2005; Yang, 2001). In an earlier paper (Syau & Lee, 2005), we extended the class of E-convex functions to E-quasiconvex functions. More recently, (Fulga and Preda, 2009) introduced a class of sets, called E-invex sets, which generalize the concept of E-convex sets. Based on the proposed E-invex sets, they introduced the class of E-preinvex functions as a generalization of E-convex and preinvex functions, and extended the

class of E-quasiconvex functions to E-prequasiinvex functions.

Recall that the lower level sets of convex functions are convex sets, and that the lower level sets of preinvex functions are invex sets. Due to these properties, a convex (resp. preinvex) programming with inequality constraints may be easily formulated. However, an E-convex function does not guarantee E-convexity of its lower level sets. Based on the concept of E-convex sets, (Chen, 2002) introduced and studied a new concept of semi-E-convex functions which guarantee E-convexity of their lower level sets.

Motivated both by earlier research works (Chen, 2002; Fulga & Preda, 2009) and by the importance of preinvexity, we introduce a class of functions called semi-E-preinvex functions which are generalizations of semi-E-convex functions. Similarly, the concept of semi-E-quasiconvex functions is also generalized to semi-E-prequasiinvex functions. Some properties of these proposed classes are studied, and sufficient conditions for a nonempty subset of the n-dimensional Euclidean space to be an E-convex or E-invex set are given. The relationship between semi-E-preinvex and E-preinvex functions is discussed. In addition, some results for the corresponding nonlinear programming problems are considered.

2. PRELIMINARIES

Let R^n denote the n-dimensional Euclidean space. Recall (Youness, 1999) that, by definition, a set $M \subseteq R^n$ is said to be E-convex if there is a mapping $E: R^n \to R^n$ such that

$$\lambda E(x) + (1 - \lambda) E(y) \in M, \ \forall x, y \in M, \ \forall \lambda \in [0, 1].$$

Let $E: R^n \to R^n$ be a given mapping. For a nonempty set $S \subseteq R^n$, let

$$E(S) = \{E(x): x \in S\}.$$

Lemma 2.1 (Youness, 1999, Proposition 2.2). Let $M \subseteq R^n$ be a nonempty E-convex set, then $E(M) \subseteq M$.

A set $K \subseteq R^n$ is said to be an invex set with respect to (w.r.t. in short) a given mapping $\eta: R^n \times R^n \to R^n$ if

$$x, y \in K, \lambda \in [0, 1] \implies y + \lambda \eta(x, y) \in K.$$

Definition 2.1 (Mohan & Neogy, 1995). Let $K \subseteq R^n$ be a nonempty invex set w.r.t. a given mapping $\eta: R^n \times R^n \to R^n$. A function $f: K \to R^1$ is said to be preinvex on K w.r.t. η if for all x, $y \in K$ and $\lambda \in [0,1]$,

$$f(y + \lambda \eta(x, y)) \leq \lambda f(x) + (1 - \lambda)f(y).$$

Throughout this section, let $\eta: R^n \times R^n \to R^n$ be a fixed mapping.

Definition 2.2 (Chen, 2002; Syau & Lee, 2005; Youness, 1999). Let $M \subseteq R^n$ be nonempty. A function $f: R^n \to R^1$ is said to be

E-convex on M if there is a mapping $E: R^n \to R^n$ such that M is an E-convex set and

$$f(\lambda E(x) + (1 - \lambda)E(y)) \leq \lambda f(E(x)) + (1 - \lambda)f(E(y))$$

for all $x, y \in M$ and $\lambda \in [0,1]$;

E-quasiconvex on M if there is a mapping $E: R^n \to R^n$ such that M is an E-convex set and

$$f(\lambda E(x) + (1 - \lambda)E(y)) \leq \max \ \{f(E(x)), f(E(y))\}$$

for all $x, y \in M$ and $\lambda \in [0,1]$; semi-E-convex on M if there is a mapping $E: R^n \to R^n$ such that M is an E-convex set and

$$f(\lambda E(x) + (1 - \lambda)E(y)) \leq \lambda f(x) + (1 - \lambda)f(y)$$

for all $x, y \in M$ and $\lambda \in [0,1]$; semi-E-quasiconvex on M if there is a mapping $E: R^n \to R^n$ such that M is an E-convex set and

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