Chapter 12 A Study of Tabu Search for Coloring Random 3-Colorable Graphs Around the Phase Transition

Jean-Philippe Hamiez Université d'Angers/LERIA, France

> **Jin-Kao Hao** Université d'Angers, France

Fred W. Glover OptTek Systems Inc., USA

ABSTRACT

The authors present an experimental investigation of tabu search (TS) to solve the 3-coloring problem (3-COL). Computational results reveal that a basic TS algorithm is able to find proper 3-colorings for random 3-colorable graphs with up to 11000 vertices and beyond when instances follow the uniform or equipartite well-known models, and up to 1500 vertices for the hardest class of flat graphs. This study also validates and reinforces some existing phase transition thresholds for 3-COL.

INTRODUCTION

Given a simple undirected graph G = (V(G), E(G)), where $V(G) = \{v_1, v_2, ..., v_n\}$ is a set of *n* vertices (*n* is usually called the "order" of *G*) and $E(G) \subset V(G) \times V(G)$ a set of *m* edges, and a set $C = \{c_1, c_2, ..., c_k\}$ of *k* colors, a *k*-coloring of *G* is any assignment of one of the *k* available colors

from *C* to every vertex in V(G). More formally, a *k*-coloring of *G* is a mapping $c: V(G) \rightarrow C$. The *k*-coloring problem (*k*-COL) is to find such a mapping (or prove that none exists) such that adjacent vertices receive different colors (called "proper" *k*-coloring). More formally, a proper *k*-coloring of *G* verifies $\{v_i, v_j\} \in E(G) \rightarrow c(v_i) \neq c(v_j)$. The tightly related optimization version of *k*-COL is the graph coloring problem (COL): Determine a

proper *k*-coloring of *G* with *k* minimum, i.e. the *chromatic number* $\chi(G)$.

k-COL is known to be *NP-complete* when $k \ge 3$ for general graphs (Garey & Johnson, 1979; Karp, 1972). It remains NP-complete even for *particular* classes of graphs, including, for instance, triangle-free graphs with maximum degree 4 (Maffray & Preissmann, 1996). Classes of graphs for which 3-COL can be decided in polynomial time are discussed, for instance, in (Alekseev et al., 2007; Kochol et al., 2003).

Another way to express the difficulty of a combinatorial search problem is to consider the phase transition phenomenon which refers to the "easy-hard-easy" transition regions where a problem goes from easy to hard, and conversely (Cheeseman et al., 1991; Dubois et al., 2001; Gent et al., 1996; Hartmann & Weigt, 2005; Hogg et al., 1996; Monasson et al., 1999), see also (Barbosa & Ferreira, 2004; Krzakała et al., 2004; Zdeborová & Krzakała, 2007) for k-COL. Various phase transition thresholds (noted τ hereafter) have been identified for some classes of random graphs. For 3-COL, τ seems to occur when the edge probability *p* is such that $2pn/3 \approx 16/3$ according to Petford & Welsh (1989) (referred as τ_w in the rest of the paper), when the mean connection degree 2m/n ≈ 5.4 (τ_c from Cheeseman et al. (1991)), when $7/n \le p \le 8/n$ (τ_h from Eiben, van der Hauw, & van Hemert, 1998), when $2m/n \approx 4.6$ (τ_{a} from Culberson & Gent, 2001), or when $p \approx 3/n + 3(n + 3)$ - 3) $(1 - 1/6^{2/n})/2n$ (τ_{ρ} from Erben (2001)). Note that τ_e and τ_w are similar to the upper bound of τ_h (8/n). τ_c and τ_g are also similar but τ_c holds only for graphs that are first transformed (before solving) using three "particular reduction operators" (Cheeseman et al., 1991). Additionally, τ_a was characterized just for equipartite graphs and $\tau_{\rm m}$ only for equipartite and uniform graphs. Henceforth, we use the terminology *outside* of τ_{μ} (or τ_{c} or τ_{a}) etc.) to indicate parameter values outside of the indicated τ setting.

This paper focuses on an experimental study of finding solution for 3-colorable random graphs

around and outside of phase transitions. We are particularly interested in two questions. First, are graphs around phase transitions really difficult to color from a practical solution point of view? Effectively, the different thresholds for phase transition have been established either theoretically or empirically. In both cases, it would be interesting to verify these thresholds by large scale computational experimentation. Notice that, except (Eiben et al., 1998), most experimental studies (see e.g., Cheeseman et al., 1991; Hogg et al., 1996) are based only on systematic backtracking search algorithms and small graphs (with no more than 200 vertices). Little is known about the behavior of a (metaheuristic-based) search algorithm on solving large and very large 3-colorable graphs.

Closely related to this first question is another interesting point: Given the phase transition phenomenon, what are the largest sizes of the graphs that can be colored in practice? Actually, the phase transition thresholds distinguish the relative hardness of instances around and outside of the thresholds. They don't tell much about whether such instances can be solved easily with a practical solution algorithm (such as tabu search) and for which problem sizes a solution is possible.

In this study, we aim to investigate these issues by studying a large range of random graphs generated according to three well-known distributions: Uniform, equipartite, and flat (see next section for more details). For the solution algorithm, we employ a simple tabu search (TS) algorithm (Glover & Laguna, 1997) which can be considered as a baseline reference for the class of metaheuristic (k-) coloring algorithms.

We report computational results on graphs with up to 11000 vertices, leading to two main findings. First, the variation of solution difficulty of random graphs around and outside of phase transition thresholds are clearly confirmed throughout the experiments: Graphs around the phase transition thresholds are actually more difficult to color than those outside of the thresholds. Second, for the three classes of graphs (uniform, equipartie and 26 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the publisher's webpage: www.igi-global.com/chapter/study-tabu-search-coloring-random/63812

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