Chapter 3
Wave Propagation in Filamental Cellular Automata

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ABSTRACT

Motivated by questions in biology and distributed computing, the authors investigate the behaviour of particular cellular automata, modelled as one-dimensional arrays of identical finite automata. They investigate what kinds of self-stabilising cooperative behaviour may be induced in terms of waves of cellular state changes along a filament of cells. The authors report the minimum requirements, in terms of numbers of states and the range of communication between automata, for this behaviour to be observed in individual filaments. They also discover that populations of growing filaments may have useful features not possessed by individual filaments, and they report the results of numerical simulations.

1. INTRODUCTION

In both the realms of nanotechnology and natural biology there is the need to study how the behaviour of individual cellular components, acting through purely local stimuli, produce patterns of coordinated behaviour in extended systems made up of large numbers of these microscopic cells. *Cellular automata* have been successfully applied to the study of natural systems (Chopard & Droz, 1998; Deutsch & Dormann, 2005; Ermentrout & Edelstein-Keshet, 1993), and we are particularly interested in the emergence of oscillating *wave patterns*. Such patterns are central to the study of systems as diverse as cellular pattern formation (Alber, *et al.*, 2002), excitable media (Greenberg & Hastings, 1978) and physiological development (Koch & Meinhardt, 1994).

This paper reports on preliminary studies of particular cellular automata, namely one-dimensional strings (*filaments*) of identical finite automata (*cells*). A *filament state* is simply the string of states of the automata reading, say, from left to right along the filament. The automata take as input states of its neighbours and, depending on

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its current state, the input determines the next state of the automaton. Working in synchronised cycles this local behaviour determines successive states of the filament. We are then particularly interested in the behaviour, over time, of the filament state. Under what conditions may it exhibit coordination of the individual cellular components? For the domains of interest that we have in mind, we are interested in the simplest of automata, in terms of numbers of states, minimal input and design details that will induce coordination of cellular activity. In particular, we mainly concentrate on automata with no more than three states. Also, we only deal with the simplest of filaments, namely those consisting of *identical* finite automata.

1.1. Previous Work

The classic synchronization problem in cellular automata is the so-called *firing squad problem* (introduced by Myhill in 1957, but not described in print until 1962 (Moore, 1962)). This concerns a line of identical finite automata ("soldiers"), each initialised to the same state (except for a single "captain" at far left). Soldiers take input only from their immediate neighbour(s), and the problem is to find a set of rules such that all soldiers enter a unique firing state at the same time. No 4-state solution to this problem exists, and the best-known solution has 6 states (Mazoyer, 1988). Although this is a classical problem, it is of tangential interest to us, because it seeks to home in on a one-off event, rather than looking for cyclical behaviour of the system. Others have been interested in similar issues. For example, Dijkstra (Dijkstra, 1974; Dijkstra, 1986) has discovered the existence of self-stabilising rings of automata. A self-stabilising system always returns to some "legitimate" configuration, no matter how it is perturbed (Burns & Pachl, 1989). Dijkstra described a solution to this problem for a ring of automata, where each machine may read the state of its two neighbours. However, Dijkstra's rings require more than one type of automaton for stable coordinated behaviour and, overall, his designs are more feasible for distributed digital computation, rather than the biological domain we have in mind. Others (Das et al., 1995; Jiminez-Morales et al., 2002) have specifically concentrated on evolving finite automata for one-dimensional cellular automata arranged in a ring. These authors, in our opinion, entertain overly complicated designs of finite automata and their interest in rings imposes unnaturally on potential biological applicability. Also, it is not always clearly explained how an evolved automata induces the observed behaviour across the ring. Previous work in this area can all be classified in terms of the topology in the state space of the system, in which the (other) topology of the finite automata connections is *static*. We introduce a new dimension of *varying* this latter topology over time (by periodically extending filaments to simulate growth, at a pace slower than it takes for the system to stabilise) and this is entirely new with potential relevance to, for example, certain biological systems.

It is partly for these reasons that we have embarked on what we hope will be a systematic study and this paper reports some early findings. Not least amongst these is the discovery that very simple finite automata may induce stable coordinated behaviour in populations of filaments when the same automata are insufficiently powerful to be self-stabilising for individual filaments. Among other results, our studies also reveal a simple so-called *clock automaton* that, for any number of states, will increment the cellular states of the filament (*modulo* the number of states) in *unison*.

2. SOME GENERAL POINTS

Formally, we define a filament as a homogenous one-dimensional cellular automaton with specific non-periodic boundary conditions; namely, the state transitions of *end* cells in the lattice recognize an "empty" state representing missing cells in their neighbourhoods. That is, both the left-hand