Chapter IV
A Complex-Valued Hopfield Neural Network:
Dynamics and Applications

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ABSTRACT

This chapter describes Complex Hopfield Neural Network (CHNN), a complex-variable version of the Hopfield neural network, which can exist in both fixed point and oscillatory modes. Memories can be stored by a complex version of Hebb's rule. In the fixed-point mode, CHNN is similar to a continuous-time Hopfield network. In the oscillatory mode, when multiple patterns are stored, the network wanders chaotically among patterns. Presence of chaos in this mode is verified by appropriate time series analysis. It is shown that adaptive connections can be used to control chaos and increase memory capacity. Electronic realization of the network in oscillatory dynamics, with fixed and adaptive connections shows an interesting tradeoff between energy expenditure and retrieval performance. It is shown how the intrinsic chaos in CHNN can be used as a mechanism for "annealing" when the network is used for solving quadratic optimization problems. The network's applicability to chaotic synchronization is described.

INTRODUCTION

Drawing important ideas from several sources, - the idea of associative memory from psychology (Kohonen, 1977), the idea of Hebbian adaptation from neurophysiology (Hebb, 1949), the idea of neuron as a thresholding device from prior modeling work (McCulloch & Pitts, 1943) etc., - Hopfield presented an elegant model of associative memory storage and retrieval in the brain (Hopfield, 1982; Hopfield, 1984). Most importantly, an original contribution of the Hopfield model is the suggestion that memories correspond to attractors of neural network dynamics. This essential insight has helped to create a whole class of "neural memories."

Since memories, by their very nature, must have certain stability, and there must be mechanisms for storage and retrieval of the same, it is reasonable to think of memories as attractors of brain dynamics. There is also some experimental evidence towards that end. But where experimental data differs from Hopfield's model memories is that brain memories are not fixed point attractors, the way Hopfield's memories are. For example, work done by Freeman and his group with mammalian olfactory cortex revealed that odors are stored as oscillatory states (Skarda & Freeman, 1987). Synchronization, an important phenomenon related to oscillations plays a significant role in information processing in the brain. It has been suggested that oscillations in visual cortex may provide an explanation for the binding problem (Gray & Singer, 1989). This result has come as experimental support to Malsburg's labeling hypothesis (von der Malsburg, 1988), which postulates that neural information processing is intimately related to the temporal relationships between the phase- and/or frequency-based "labels" of oscillating cell assemblies. All these phenomena cannot be captured by neural models that exhibit only fixed-point behavior.

Neural models in which memories can be stored as oscillations have been proposed before. Abbot (1990) studied a network of oscillating neurons in which binary patterns can be stored as phase relationships between individual oscillators. The Hopfield model too can exhibit limit cycles and chaos but only when the symmetry condition on weights is relaxed (Sompolinsky, Crisanti & Sommers, 1988; Albers, Sprott, & Dechert, 1998). When the symmetry condition is violated, the Hebb's rule for storing patterns is no more valid in general, except in special cases like storing short sequences.

BACKGROUND

It has been shown that by extending Hopfield's real-valued model to complex –variable domain, it is possible to preserve the symmetric Hebbian synapses, while permitting the network to have oscillatory states (Chakravarthy & Ghosh, 1996). Pioneering work on complex-valued versions of Hopfield network was done by Hirose (1992). Other studies in the area of complex neural networks include complex backpropagation algorithm for training complex feedforward networks (Leung & Haykin, 1991; Nitta, 1997) and a similar extension for complex-valued recurrent neural networks (Mandic & Goh, 2004). For a comprehensive review of complex neural models the reader may consult (Hirose, 2003).

In the present chapter, we discuss the properties and applications of a particular complex neural network model viz., the complex Hopfield neural network (CHNN). The chapter is organized as follows. We begin with a brief review of the original real-valued Hopfield network, which is followed by a plausible biological interpretation of the complex state of a neuron in the next Section. The model equations of CHNN are presented in the subsequent Section, which is followed by a Section that presents learning mechanisms. Learning can be a one-shot affair where the weights are pre-calculated by a complex Hebb's rule. Or learning can occur continuously, with weight update described by differential equations. The following section describes the two modes in which the proposed network operates: 1) fixed point mode and 2) oscillatory mode. In the subsequent two sections, associative memory function of CHNN in the two modes is described. It will be shown that memory capacity of the network in oscillatory mode is very poor. However, it will be also shown, in the subsequent Section, that by allowing the weights to adapt dynamically, *even during retrieval*, memory capacity can be enhanced significantly even in the oscillatory mode. The next Section presents an electronic realization of the model. The following Section describes application of CHNN for quadratic optimization. The chaotic dynamics of the network in oscillatory mode is exploited as a mechanism for avoiding getting stuck in a local minimum. An application of CHNN for chaotic synchronization useful for secure communications is discussed in the following Section. An overview of the work and challenges for future are discussed in the final Section.

THE REAL-VALUED HOPFIELD NETWORK

In a landmark paper, Hopfield (1982) proposed a neural network implementation of an associative memory in which binary patterns can be stored and retrieved. The McCulloch-Pitts (McCulloch & Pitts, 1943) binary neuron is used in this network. In the Hopfield's neural network each neuron is connected to every other neuron through weights $T = \{ T_{jk} \}$, where T_{jk} is the weight connecting j'th and k'th neuron. Each neuron receives inputs from all

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