

# Chapter VI

## Models of Complex-Valued Hopfield-Type Neural Networks and Their Dynamics

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### ABSTRACT

*This chapter presents models of fully connected complex-valued neural networks which are complex-valued extension of Hopfield-type neural networks and discusses methods to study their dynamics. In particular the authors investigate existence conditions of energy functions for complex-valued Hopfield-type neural networks. Emphasized are the properties of the activation functions which assure the existence of an energy function for the networks. As an application of the energy function, qualitative analysis of the network by utilizing the energy function is shown and a synthesis method of complex-valued associative memories is discussed.*

### INTRODUCTION

In recent years, there have been increasing research interests of artificial neural networks and many efforts have been made on applications of neural networks to various fields. As applications of the neural networks spread more widely, developing neural network models which can directly deal with complex numbers is desired in various fields. Several models of complex-valued neural networks have been proposed and their abilities of information processing have been investigated.

The purpose of this chapter is to present models of fully connected complex-valued neural networks which are complex-valued extension of Hopfield-type neural networks and to discuss methods to investigate their dynamics. In particular, we investigate existence conditions of energy functions and propose a function as an energy function for the complex-valued neural networks. In the complex region there are several possibilities in choosing an activation function because of a variety of complex functions. Based on the existence condition we investigate the properties of the complex functions which assure the existence of an energy function and discuss about how to find them. Several classes of complex functions which are widely used as activation functions in the models of complex-valued neural networks proposed so far are considered.

Once energy functions are constructed for neural networks, they are expected to be applied to various problems of various fields such as qualitative analysis of the neural networks, synthesis of associative memories and several optimization problems, similar to the real-valued ones. As applications, this chapter presents qualitative analysis of dynamics of the complex-valued neural networks by using the energy function. Furthermore a synthesis method of complex-valued associative memories by utilizing the analysis results is discussed.

In the following, the imaginary unit is denoted by  $i$  ( $i^2 = -1$ ). The set of complex (real) numbers is denoted by  $\mathbb{C}$  ( $\mathbb{R}$ ). The  $n$ -dimensional complex (real) space is denoted by  $\mathbb{C}^n$  ( $\mathbb{R}^n$ ) and the set of  $n \times m$  complex (real) matrices is denoted by  $\mathbb{C}^{n \times m}$  ( $\mathbb{R}^{n \times m}$ ). For  $A \in \mathbb{C}^{n \times m}$  ( $a \in \mathbb{C}^n$ ), its real and imaginary parts are denoted by  $A^R$  ( $a^R$ ) and  $A^I$  ( $a^I$ ), respectively.

## BACKGROUND

It is well known that one of the pioneering works that triggered the research interests of neural networks in the last two decades is the proposal of models for neural networks by J. J. Hopfield (Hopfield, 1984; Hopfield & Tank, 1985), which are fully connected recurrent neural networks. He introduced the idea of an energy function to formulate a way of understanding the computation performed by dynamics of fully connected neural networks and showed that a combinatorial optimization problem can be solved by the neural networks. The neural network models proposed by Hopfield are called Hopfield type neural networks and by using concept of energy functions they have been applied to various problems such as qualitative analysis of neural networks, synthesis of associative memories, optimization problems etc. ever since. It is, therefore, of great interest to develop models of complex-valued neural network of Hopfield type and to investigate their dynamics.

In extending the discussions on real-valued neural networks to the complex plane it is important to note the following. One of the important factors to characterize behavior of a complex-valued neural network is its activation function which is a nonlinear complex function. In the real-valued neural networks, the activation is usually chosen to be a smooth and bounded function such as a sigmoidal function. In the complex region, however, there are several possibilities in choosing an activation function because of a variety of complex functions. It is expected, therefore, that complex-valued neural networks exhibit wide variety of dynamics depending on which type of complex functions is used as activation functions and their applications spread widely by using their wide variety of dynamics.

## HOPFIELD TYPE NEURAL NETWORKS

J. J. Hopfield proposed discrete- and continuous-time models of fully connected recurrent neural networks and introduced the idea of an energy function to formulate a way of understanding the computation performed by their dynamics, which triggered the research interests of neural networks. In this chapter we consider the continuous-time Hopfield type neural network, which is implemented by an electric circuit shown in Fig. 1 (Hopfield, 1984; Hopfield & Tank, 1985).

The circuit consists of  $n$  nonlinear amplifiers interconnected by an RC (resistor-capacitor) network, and conductances and ideal current sources. Each amplifier provide an output voltage  $x_j$  given by  $f(u_j)$ , where  $u_j$  is the input voltage and  $f$  is a nonlinear activation function. For each amplifier, it contains an inverting amplifier whose output is  $-x_j$  which permits a choice of the sign of the amplifier. The outputs  $x_j$  and  $-x_j$  are usually provided by two output terminals of the same operational amplifier circuit. The pair of nonlinear amplifiers with an RC network is referred to as a “neuron” and the RC network partially defines the time constant of the neuron and provides for integrative analog summation of the synaptic input currents from other neurons in the network. A synapse between two neurons is defined by a conductance  $T_{jk}$  which connects one of the two outputs ( $x_k$  or  $-x_k$ ) of amplifier  $k$  to the input of amplifier  $j$  and this connection is made with a resistor of value  $R_{jk} = 1/|T_{jk}|$ . As shown in Fig. 1, the circuit included an externally supplied input current  $I_j$  for each neuron, which represents an external input signal (or bias) to neuron  $j$ .

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