# Chapter 17 Composition of Functional Petri Nets

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## ABSTRACT

Functional Petri nets and subnets are introduced and studied for the purpose of speed-up of Petri nets analysis with algebraic methods. The authors show that any functional subnet may be generated by a composition of minimal functional subnets. They propose two ways to decompose a Petri net: via logical equations solution and with an ad-hoc algorithm, whose complexity is polynomial. Then properties of functional subnets are studied. The authors show that linear invariants of a Petri net may be computed from invariants of its functional subnets; similar results also hold for the fundamental equation of Petri nets. A technique for Petri nets analysis using composition of functional subnets is also introduced and studied. The authors show that composition-based calculation of invariants and solutions of fundamental equation provides a significant speed-up of computations. For an additional speed-up, they propose a sequential composition of functional subnets. Sequential composition is formalised in the terms of graph theory and was named the optimal collapse of a weighted graph. At last, the authors apply the introduced technique to the analysis of Petri net models of such well-known networking protocols as ECMA, TCP, BGP.

#### INTRODUCTION

Approach of decomposition and following composition is an implementation of an ancient principle "divide and sway" to the systems' analysis. Since the majority of Petri net analysis methods are exponential either in space or in time, such auxiliary techniques as decomposition and reduction become attractive promising a certain speed-up of the analysis process. It is significant when dealing with real-life models of manufacturing systems counting thousands nodes. Such a speed up can shift a task from the grade of unfeasible to the grade of solvable in admissible time.

There were taken lots of attempts to decompose Petri net bringing some advantages but introduced in this chapter concept of functional subnet is the most strict and clear requiring to solve a single composition system which allows a considerable speed-up of Petri nets analysis process. Program implementation of Petri nets structural analysis based on composition of functional subnets was done in the form of plug-ins Deborah and Adriana for modelling system Tina. They were tested on a series of networking protocols' models that acknowledges the practical value of the technique.

The plot of the chapter is the following. In Section 2, we introduce and discuss the concepts of functional Petri net and functional subnet. In Section 3, we study properties of functional subnets and consider the representation of decomposition with a net of functional subnets and a graph of decomposition. In Section 4, two different ways of decomposition are studied: with the aid of logical equations and using an ad-hoc algorithm of linear complexity on size of the net. In Section 5, we describe the technique of composition-based calculation of linear invariants and show the exponential speed-up of calculations. In Section 6, we obtain analogous results for fundamental equation of Petri net. In Section 7, we propose to use sequential composition to provide an additional speed-up during solution of systems for contact places. Two ways of sequential composition using subgraphs and edges are discussed. Then we study in detail the edge sequential composition formalized as the task of edge collapse of the weighted graph. In Section 8, we present examples of invariants calculation via simultaneous and sequential composition of functional subnets for Petri net models of networking protocols ECMA, TCP, BGP.

## BACKGROUND

Linear algebra methods (Diaz, 2001; Murata, 1989; Reisig, 1982) based on state equation and invariants are a powerful tool for Petri nets analysis. But to find linear invariants and to solve the fundamental equation of a Petri net we have to solve linear Diophantine systems in nonnegative

integer numbers. All known methods of such systems solution (Colom & Silva, 1990; Contejean & Ajili, 1997; Kryviy, 1999; Martinez & Silva, 1982; Schrejver, 1987; Toudic, 1982) possess exponential complexity with respect to space. It makes the analysis of large-scale models practically unfeasible and requires searching of new techniques, which provide essential speed-up of computations.

Two basic approaches were suggested (Berthelot, 1987) to handle large-scale nets: decomposition and reduction. Implementations of these approaches have been designed in the different concrete ways. Moreover, decomposition and reduction are applied not only to nets but to state space also. Reduction provides a set of rules for decreasing the dimension of net preserving its properties. Then usual analysis methods are applied to reduced net.

Decomposition and composition (Singh & Titly, 1986) are abstraction-based methods successfully applied in different fields of science and engineering. On the one hand the majority of artificial systems are composed out of their components and this process is hierarchical. So there is a decomposition of systems provided by the rules of their construction and the set of their components and elements (Cortadella et al., 2002; Girault & Valk, 2003; Jensen, 1997; Juhas et al., 2004). The simplest way assumes the usage of such decomposition. If we know the properties of components and use special rules of composition (synthesis) preserving properties, then we construct an ideal system (Juan et al., 1998; Kotov, 1984). But unfortunately, it is not a prevailing case for real-life objects. On the other hand, the goals of concrete analysis often require tricky decomposition. Decomposition is justified if there are techniques allowing the determination of systems' properties on the base of properties of their components. Thus, decomposition methods always assume the following composition of a system.

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