Chapter 6

Measuring and Explaining Economic Inequality:
An Extension of the Gini Coefficient

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ABSTRACT

This chapter proposes a new class of inequality indices based on the Gini coefficient (or index). The properties of the indices are studied and are found to be regular, relative, and to satisfy the Pigou-Dalton transfer principle. A subgroup decomposition is performed, and the method is found to be similar to the one used by Dagum when decomposing the Gini index. The theoretical results are illustrated by case studies, using Cameroonian data.

1. INTRODUCTION

Research on the measurement of economic inequality is dominated by the Gini index (or coefficient) and the entropy family of indices. Many studies have been devoted to the properties of these two categories of indices. Since the early works of Gini (1916), the Gini index has been studied by many authors, so that nowadays it lends itself to axiomatic characterisation and at least to two kinds of generalisations (Chotikapanich and Griffits, 2001; Yitzhaki, 1983). The index's decomposition into sub-groups, which previously has not been very satisfactory, has been improved by the recent works of Dagum (1997a, 1997b) who proposes a new approach for solving the problem. More recently, Mussard (2004) proposed a simultaneous decomposition of the Gini index into sub-groups and sources of income, and so on.

The present study is in keeping with this area of research, which it attempts to extend. We propose a family of inequality indices, denoted $I_G^{(\alpha)}$, which generalise the Gini index, and which intersect the entropy family through the coefficient of variation squared. We analyse the axiomatic properties of our class of indices and show that it is a class of relative, regular indices, which satisfy the Pigou-Dalton transfer principle. We study the consequences of a transfer from a richer to a poorer individual and show that the effect of such a transfer is maximal at a central value of the income distribution, which we define.

Next we show that $I_G^{(\alpha)}$ lends itself to decomposition into sub-groups. The decomposition proposed is a generalisation of Dagum's decomposition of the Gini index.

The remainder of the chapter is organized as follows: In section 2, we present notation and preliminaries. In section 3, we define the index $I_G^{(\alpha)}$ and analyze its properties. Decomposition of the proposed index into sub-groups is undertaken in section 4. Section 5 analyzes the particular case of $\alpha = 2$, corresponding to coefficient of variation squared, which also belongs to the family of entropy indices. Finally, section 6 concludes the chapter.

2. NOTATION AND PRELIMINARIES

In this chapter, $P = \{1, 2, 3, ..., i, ...n\}$ is a population with n members. X is a positive variable defined in P and represents an income source distribution between the n members of P. We denote by $x_1, x_2, x_3, \cdots, x_i, \cdots, x_n$ the values of X on the n members of P respectively. We assume that P is partitioned into K subpopulations $P_1, P_2, P_3, \cdots, P_h, \cdots, P_K$ with, respectively, $n_1, n_2,, n_3, \cdots, n_h, \cdots, n_K$, $\sum_{h=1}^{K} n_h = n$, members. The value of X on member number i of P_h is written as x_{hi}. The restriction of X in P_h is written X_h; $\mu \left(\mu_h \right)$ is the mean of X in P (in P_h) and $Var(X)$ $\left(Var(X_h) \right)$ represents the variance of X in P (in P_h). Also, $CV^2 \left(X \right) \left(CV^2 \left(X_h \right) \right)$ is the square of the coefficient of variation of X in $P \left(in\ P_h \right)$:

$$CV^2 \left(X \right) = \frac{Var\left(X \right)}{\mu^2}$$

and

$$CV^2(X_h) = \frac{Var(X_h)}{\mu_h^2}.$$

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i, \ \mu_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}, \ \mu = \frac{1}{n} \sum_{h=1}^{K} n_h \mu_h \tag{1}$$

For any real number α, we define the two real functions:

$$D_\alpha \left(x \right) = \sum_{x_i \leq x} \left(x - x_i \right)^\alpha -$$
$$\sum_{x_i \geq x} \left(x - x_i \right)^\alpha = \sum_{x_i \leq x} \left| x - x_i \right|^\alpha - \sum_{x_i \geq x} \left| x - x_i \right|^\alpha, \tag{2}$$

and

$$H_\alpha \left(x \right) = \sum_{x_i \leq x} \left(x - x_i \right)^\alpha -$$
$$\sum_{x_i \geq x} \left(x - x_i \right)^\alpha = \sum_{i=1}^{n} \left| x - x_i \right|^\alpha, \tag{3}$$

where, $D_\alpha(x)$ represents the sum of differentials (to the power α) relative to x of the income less than x minus the sum of differentials relative to x of the incomes which are greater than it. $H_\alpha(x)$ represents the sum of differentials to the power α, relative to x of all the incomes of the population.

2.1. Properties of $D_\alpha \left(x \right)$ and $H_\alpha \left(x \right)$ and their Relationships

2.1.1. Properties of $D_\alpha(x)$

1. $\alpha = 0$
   a. $\forall x \in R, D_0(x) = $ (Number of $x_i \leq x$) -(Number of $x_i \geq x$)
   b.   If we assume $x_1 < x_2 < x_3 < ... < x_n$, then

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