# Chapter II
# Robust Adaptive Beamforming

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## INTRODUCTION

In this chapter, we first review the background, basic principle and structure of adaptive beamformers. Since there are many robust adaptive beamforming methods proposed in literature, for easy understanding, we organize them into two categories from the mathematical point of view: one is based on quadratic optimization with linear and nonlinear constraints; the another one is max-min optimization with linear and nonlinear constraints. With the max-min optimization technique, the state-of-the-art robust adaptive beamformers are derived. Theoretical analysis and numerical results are presented to show their superior performance.

## BACKGROUND

The array signal processing has been studied for some decades as an attractive method for signal detection and estimation in hash environment. An array of sensors can be flexibly configured to exploit spatial and temporal characteristics of signal and noise and has many advantages over single sensor. It has many applications in radar, radio astronomy, sonar, wireless communication, seismology, speech acquisition, medical diagnosis and treatment (Tsoulos, 2001) (Krim & Viberg, 1996) (Van Veen & Buckley, 1988), etc.

There are two kinds of array beamformers: fixed beamformer and adaptive beamformer. The weight of fixed beamformer is pre-designed and it does not change in applications. The adaptive beamformer automatically adjusts its weight according to some criteria. It significantly outperforms the fixed beamformer in noise and interference suppression. A typical representative is the linearly constrained minimum variance (LCMV) beamformer (Compton, 1988) (Hudson, 1981) (Johnson & Dudgeon, 1993) (Monzingo & Miller, 1980) (Naidu, 2001). A famous representative of the LCMV is the Capon beamformer (Capon, 1969). In ideal cases, the Capon beamformer has high performance in interference and noise suppression provided that the array steering vector (ASV) is known. However, the ideal assumptions of adaptive beamformer may be violated in practical applications (Vural, 1979) (Jablon, 1986a) (Jablon, 1986b) (Cox, Zeskind, & Owen, 1988) (Chang & Yeh, 1993). The performance of the adaptive beamformers highly degrades when there are array imperfections such as steering direction error, time delay error, phase errors of the array sensors, multipath propagation effects, wavefront distortions. This is known as the target signal cancellation problem. Tremendous work has been done to improve the robustness of adaptive beamformer (Nunn, 1983) (Er & Cantoni, 1983) (Buckley & Griffiths, 1986) (Er & Cantoni, 1986c) (Er, 1988) (Thng, Cantoni, & Leung, 1993) (Thng, Cantoni, & Leung, 1995) (Zhang & Thng, 2002) (Er & Cantoni, 1986b) (Er & Cantoni, 1990) (Cox, Zeskind, & Owen, 1987) (Vorobyov, Gershman, & Luo, 2003) (Lorenz & Boyd, 2005) (Li, Stoica, & Wang, 2003) (Stoica, Wang, & Li, 2003) (Affes & Grenier, 1997) (Er & Ng, 1994).

To overcome the problem of target signal cancellation caused by the steering direction error, multiple-point constraints (Hudson, 1981)(Nunn, 1983) were introduced in adaptive array. The idea of this approach is intuitive. With multiple gain constraints at different directions in the vicinity of the assumed one, the array processor becomes robust in the region where constraints are imposed. However, the available number of constraints is limited because the constraints consume the degrees of freedom (DOFs) of array processor for interference suppression.

Another class of solution is to introduce the derivative constraints into the array processor (Hudson, 1981) (Er & Cantoni, 1983) (Buckley & Griffiths, 1986) (Er & Cantoni, 1986c) (Er, 1988) (Thng et al., 1993) (Thng et al., 1995) (Zhang & Thng, 2002). With the derivative constraints, the array response is almost flat in the vicinity of target direction. The beamformer has widened beamwidth in the target direction. With a small steering direction error, the beamformer does not cancel the target signal. However, the widened beamwidth is achieved at the cost of reduced capability in interference suppression because the additional derivative constraints consume the DOFs of beamformer. Derivative constraints can be used to obtain not only a flat response of array processor, but also a flat null in the assumed signal direction in blocking matrix design (Fudge, 1996).

A new set of constraints for robust array processor against the steering error was also proposed in (Er & Cantoni, 1986b)(Er & Cantoni, 1990). The idea is to minimize the weighted mean square deviation between the desired array response and the response of the processor over the variations in parameters, such as the steering error, the phase errors and the array geometry error, etc. Although the constraints derived by this approach are quadratic (Er, 1985), a set of linear constraints was derived approximately (Er & Cantoni, 1986b). In the approximation of quadratic constraints to linear constraints, a problem arises that how many constraints should be selected. A method to determine the number of necessary linear constraints and to select the constraints was proposed in (Er & Cantoni, 1986b).

Techniques restraining excess coefficients growth were also proposed in array processor to achieve robust performance. When array processor cancels target signal, the norm of the filter coefficients grows to a large value beyond the normal value for noise and interference suppression. In (Cox et al., 1987), an inequality constraint is imposed on the coefficients norm of adaptive beamformer to limit the growth of tap coefficients. The excess coefficients growth problem can also be solved by using noise injection method (Jablon, 1986a). Artificially generated noise is added to reference signals of adaptive filters. Although the artificial noise causes estimation errors in the beamformer coefficients, it prevents tap coefficients from growing excessively, resulting in robustness against array imperfections. A similar approach called the leaky least mean square (LMS) algorithm can also be used (Claesson & Nordholm, 1992) for this purpose.

Other robust beamforming methods include the calibration based approaches (Fudge & Linebarger, 1994). The calibration can generally eliminate the inherent error of the array processor, such as geometry error, sensor response error, etc. However, it cannot eliminate the dynamic errors, such as the steering error when the source is moving in a vicinity of the assumed direction. Target tracking methods (Affes & Grenier, 1997)(Er & Ng, 1994) were introduced in array processor so that the look direction is steered to the continuously estimated direction-of-arrival (DOA). One problem is that this method may mistrack to the interference in the absence of target signal unless some other methods are used to limit the tracking region.

The robust beamforming methods discussed above solve part of robust beamforming problems. More research works still need to be carried out, especially in real applications. In (Er & Ng, 1994), a new approach was proposed for robust beamforming in the presence of steering direction error. It iteratively searches for the optimal direction by maximizing the mean output power of the Capon beamformer using first-order Taylor series approximation in terms of steering direction error. This method does not suffer from performance loss in interference/noise suppression. However, its performance degrades when there exist multiple errors, such as the steering direction error, the array geometry error and the array sensor phase error, because the array steering vector in (Er & Ng, 1994) is assumed to be a vector function of steering direction only. When multiple imperfections exist, the assumed model of the ASV is violated. In (Yu & Er, 2006a) (Yu, 2006), a new model of the ASV is adopted. All of these array imperfections are modeled as general phase errors (GPEs).