Chapter V
Random Array Theory and Collaborative Beamforming

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ABSTRACT

In wireless sensor networks, the sensor nodes are often randomly situated, and each node is likely to be equipped with a single antenna. If these sensor nodes are able to synchronize, it is possible to beamform by considering sensor nodes as a random array of antennas. Using probabilistic arguments, it can be shown that random arrays formed by dispersive sensors can form nice beampatterns with a sharp main lobe and low sidelobe levels. This chapter reviews the probabilistic analysis of linear random arrays, which dates back to the early work of Y. T. Lo (1964), and then discusses recent work on the statistical analysis of two-dimensional random arrays originally derived in the framework of wireless sensor networks.

INTRODUCTION

Wireless sensor networks have recently attracted much attention in the communication engineering community (e.g., Yao, 1998). In many such networks, battery powered communicating nodes, each equipped with a single antenna, are distributed randomly. These nodes not only gather information but also communicate over a wireless channel. Some nodes may relay the received information to nearby nodes or directly transmit to the destination (or fusion center). In order to deliver information to the fusion center, provided neighboring nodes possess the same information and are able to synchronize, it is also possible for these nodes to beamform collaboratively. We refer to this kind of distributed beamforming as *collaborative beamforming* (Ochiai, 2005).

In wireless communications, beamforming enables an efficient implementation of space-division multiple access (SDMA), which has the potential to significantly increase communication rates over multiple access channels. SDMA by means of collaborative beamforming is also a powerful approach in the framework of wireless *ad hoc* sensor networks where several clusters operate asynchronously.

Let us consider the scenario illustrated in Figure 1, where the sensor nodes in two separate clusters A and B are communicating with respective distant fusion centers located in different directions. In this case, intra-cluster communication among sensor nodes for information sharing may be realized by low-cost short distance broadcast-type communication. Therefore, the main challenge is fair channel allocation for long distance communication links between the clusters and the fusion centers. Since synchronization among nodes in different clusters may not be easily established, random access is commonly used. With limited available spectrum resources, however, random access typically requires additional overhead such as collision detection. Furthermore, an increase in the number of communicating clusters considerably reduces the overall throughput.

On the other hand, with collaborative beamforming depicted in Figure 1, cognition of the other communicating clusters is not necessary, provided that the direction of the fusion center is different. Due to the random nature of *ad hoc* sensor networks, it is highly unlikely that the sensor nodes in the distinct clusters are transmitting in the same direction. Therefore, collaborative beamforming has the potential to become a low-cost SDMA implementation.

A question that arises in this scenario is whether or not collaborative sensors can form a nice beampattern. Since the distribution of the sensor nodes is typically random by nature, it is reasonable to consider arrays formed by the sensor nodes as random arrays and to treat them using probabilistic arguments.

Probabilistic analysis of antenna arrays dates back to the early work of Lo (1964), who first developed a comprehensive theory of linear random arrays. Using statistical arguments, Lo showed that a random array can form a nice average beampattern without the major grating lobes that are observed in a typical periodic array. He also derived the distribution of the beampattern based on a Gaussian approximation. Later, Steinberg (1972), Agrawal & Lo (1972), as well as Donvito & Kassam (1979) analyzed the distribution of the maximum of the sidelobe peaks associated with linear random arrays. An excellent overview and analytical treatment of linear random arrays can be found in (Steinberg, 1976).

With applications to wireless sensor networks in mind, in this chapter, we consider the beampatterns of two-dimensional (planar) phased arrays randomly distributed over a disk of a given radius. Much of the mathematical detail of this analysis can be found in our recent work (Ochiai, 2005) and here we focus mainly on the significance of the obtained results on the probabilistic distribution of such beampatterns. Upon studying this work, we notice that whereas the theories and optimizations of unequally-spaced arrays, both linear and planar, are prolific in the literature (e.g., Ishimaru, 1962; Bar-Ness, 1984; Leahy, 1991), there are relatively few publications on the subject of statistical properties of planar random arrays in the context of communication engineering applications (e.g., Fante, 1991). Nevertheless, the theory of two-dimensional random arrays has found diverse applications. Optimization issues of random arrays have been studied by Holm, Elgetun, & Dahl (1997) in the framework of ultrasound imaging. Kook, Davies, & Bolton (2002) analyze the statistical distribution of two dimensional microphone arrays. More recently, the statistical behavior of more complex arrays of random subarrays has been studied by Kerby & Bernhard (2006).

This chapter is organized as follows. The subsequent section is devoted to the description of assumptions and mathematical models in the framework of wireless sensor networks. We then study statistical properties of the beampatterns of random arrays uniformly distributed over a disk of a given radius. Specifically, we evalute the average beampattern,

Figure 1. Collaborative beamforming in ad hoc wireless sensor networks