Chapter XV
Detection Based on Relaxation in MIMO Systems

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ABSTRACT

This chapter takes a closer look at a class of MIMO detention methods, collectively referred to as relaxation detectors. These detectors provide computationally advantageous alternatives to the optimal maximum likelihood detector. Previous analysis of relaxation detectors have mainly focused on the implementation aspects, while resorting to Monte Carlo simulations when it comes to investigating their performance in terms of error probability. The objective of this chapter is to illustrate how the performance of any detector in this class can be readily quantified thought its diversity gain when applied to an i.i.d. Rayleigh fading channel, and to show that the diversity gain is often surprisingly simple to derive based on the geometrical properties of the detector.

INTRODUCTION

A central component of wireless multiple input-multiple output (MIMO) systems is the symbol detector or demodulator where the receiver produces estimates of the symbols (or bits) transmitted over the MIMO channel given a set of received signals and an estimate of the channel state. However, unlike their single input-single output (SISO) equivalents, naive implementations of optimal detectors for MIMO channels often tend to be prohibitively computationally complex. This is partially due to the fact that in MIMO channels, each output signal tends to be influenced by *all* input signals. The nature of this influence is determined by the channel fading which is not a priori known.

The MIMO channel is frequently modeled in vector/matrix form according to

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v} \tag{1}$$

where $\mathbf{y}$ is the vector of *received signals* (after matched filtering and sampling), where $\mathbf{s}$ is the vector of *transmitted symbols* drawn from some constellation alphabet $\mathcal{S}$, where $\mathbf{v}$ is additive *noise*, and where $\mathbf{H}$ is the *channel matrix* modeling the input-output relation of the MIMO channel. In the particular case of the narrow-band multiple antenna channel with spatial multiplexing across antennas, the elements of $\mathbf{H}$ would have the physical interpretation of baseband equivalent complex gains between the transmitting and receiving antennas (Tse & Viswanath, 2005). The model in itself is however more general and essentially applicable to any scenario where a group of symbols are linearly modulated and transmitted over a linear channel (Barbosa, 1989). In some cases, the channel matrix will have special structure that is exploitable in the transmission and detection process. However, in the multiple antenna MIMO scenario the matrix, $\mathbf{H}$, is often unstructured and this calls for general detection methods.

Under the assumption of uncorrelated Gaussian noise and that the receiver has access to both $\mathbf{H}$ as well as $\mathbf{y}$ the maximum likelihood (ML) detector of $\mathbf{s}$ can be expressed according to

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\min_{\hat{\mathbf{s}}\in\mathcal{S}^m} \| \mathbf{y} - \mathbf{H}\hat{\mathbf{s}} \|^2 . \tag{2}$$

The ML detector thus amounts to, among all possible noise-free hypotheses $\mathbf{Hs}$, finding the one which most closely matches the vector of received signals and provides the smallest possible probability of detection error. Unfortunately, implementing the ML detector requires solving an constrained minimization problem which is computationally difficult. In fact, it can be shown that (2) is NP-hard for general $\mathbf{H}$ and $\mathbf{y}$ (Verdu, 1989) which implies that there are no known polynomial time algorithms for its solution. This has generated a large body of research into optimal and sub-optimal implementations of the ML detector. Part of this research effort has been devoted to the development of a class of detectors which may collectively bereferred to as relaxation detectors (Yener et. al., 2002; Thoen et. al., 2003; Cui et. al. 2005; Cui & Tellembura, 2006). The literature on these detectors has previously focused mainly on implementation aspects. Specifically, in the case of the *convex* relaxations studied in (Yener et. al., 2002; Thoen et. al., 2003), there are well known efficient implementations based on convex optimization (Boyd & Vandenberghe, 2004). This makes the relaxation detectors appealing from a computational complexity point of view. However, the error probability performance of these detectors is traditionally investigated through Monte Carlo simulations while the general detection performance is still not well understood.

The purpose of this chapter is to introduce some analytic tools to study the detection performance of these detectors without having to resort to simulations. In particular, we will show how ideas which are very similar to the pairwise error probability (PEP) analysis of traditional detectors may also be successfully applied to the study of the relaxation detectors. To this end, we shall throughout the chapter adopt some rather simplistic modeling assumptions regarding the quantities in (1) to enable a uniform and comprehensive treatment of the performance of the relaxation detectors while attempting to avoid a range of technical difficulties. We shall assume that $\mathbf{y}\in\mathbb{R}^n$, $\mathbf{H}\in\mathbb{R}^{n\times m}$, $\mathbf{s}\in\mathcal{S}^m\subset\mathbb{R}^m$ and $\mathbf{v}\in\mathbb{R}^n$ are real valued quantities. This is in contrast most literature on MIMO communications where a complex valued model is typically used but since most relaxation detectors are more easily treated under such an assumption we find it preferable. It should also be noted that in many cases (although with a few exceptions) the extension to the complex case is straightforward. In fact, whenever the complex symbol constellation is separable (i.e., M-QAM) the complex model can be written on an equivalent real valued form. We shall further assume that the components of $\mathbf{v}$ and $\mathbf{H}$ are zero-mean, Gaussian distributed, with variance σ^2 and n^{-1} respectively, i.e., $\mathbf{v}\sim N(0,\sigma^2\mathbf{I}_n)$ and $\mathrm{vec}(\mathbf{H})\sim N(0,n^{-1}\mathbf{I}_{nm})$. The performance of the detectors will be addressed by averaging over the realization of $\mathbf{H}$ and $\mathbf{v}$. We will also for simplicity only consider the case of a binary constellation, i.e., $\mathcal{S}=\{\pm1\}$.With these assumptions the signal to noise ratio (SNR), herein denoted ρ, is defined according to

$$\rho \equiv \frac{E\left\{\| \mathbf{Hs} \|^2\right\}}{m\sigma^2} = \frac{1}{\sigma^2}. \tag{3}$$

We will assume that $n \ge m$ which implies that $\mathbf{H}^T\mathbf{H}$ is invertible with probability one and, based on (3), for the most part use ρ^{-1} in place of σ^2 in the analysis.

Optimal and Sub-Optimal Detection

As stated in the introduction, the ML detector is given by

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\min_{\hat{\mathbf{s}}\in\mathcal{S}^m} \| \mathbf{y} - \mathbf{H}\hat{\mathbf{s}} \|^2 \tag{4}$$