Chapter 29
Optimal Design of Three–Link Planar Manipulators Using Grashof's Criterion

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ABSTRACT

The design of robotic manipulators is dictated by a set of pre-determined task descriptions and performance parameters. These performance parameters are often defined in terms of workspace dexterity, manipulability, and accuracy. Many serial manipulator applications require that the manipulator have full dexterity about a work piece or a pre-defined trajectory, that is, to approach the given point within the workspace with all possible orientations about that point. Grashof's criterion defines the mobility of four-link closed chain mechanisms in relation to its link lengths. A simple assumption can convert a three-link serial manipulator into a four-link closed chain so that its mobility can be studied using Grashof's criterion. With the help of Grashof's criterion, it is possible not only to predict and simulate the mobility of a manipulator during its design, but also to map and identify the fully-dexterous regions within its workspace. Mapping of the dexterous workspace is helpful in efficient task placement and path planning. Next, the authors propose a simple algorithm using Grashof's criterion for determining the optimal link lengths of a three-link manipulator, in order to achieve full dexterity at the desired regions of the workspace. Finally, the authors test the generated design by applying joint angle limitations.

INTRODUCTION

The problem of designing an optimal manipulator configuration is very complex, as the equations governing the motion of the end-effector in the workspace are both non-linear and complex, often

having no closed solutions. Prototyping methods such as kinematic synthesis and numerical optimization are complex and very time consuming. The inherent complexity of kinematic synthesis has helped to make a strong case for rapid prototyping methods in which manipulators are designed with very specific performance requirements or tasks point specifications. Rapid prototyping

allows designers to spend more time on design, simulation and evaluation of different manipulator configurations instead of solving mathematical models describing kinematics chains.

The study of mobility of closed chain mechanisms has interested researchers for a very long time. Understanding the mobility of chain mechanisms in relation to their link lengths can help us to design better and highly dexterous manipulators. In 1833, Grashof first introduced a simple rule to understand the mobility of four-link mechanisms [6]. This rule, commonly known as the Grashof's theorem, helps analyze the rotatability of links in a closed four-bar mechanism. This was further extended by Paul (1979), who introduced an inequality into the Grashof's theorem and proved that Grashof's criterion is both a necessary and sufficient condition for the existence of a crank in the four-bar mechanism (Chang, Lin, & Wu, 2005).

Researchers have applied Grashof's criterion to understand and study the workspace mobility of both closed and open chain planar mechanisms. Barker (1985), using Grashof's criterion, classified four-bar planar mechanisms based on their mobility. Grashof's criterion was applied to the study of three-link planar mechanism by Li and Dai (2009). Furthermore, they developed equations for the orientation angle and presented a simple program to analyze the orientation angle for a manipulator, given the link parameters. The mobility and orientation of open chain mechanisms can also be analyzed using Grashof's criterion. Dai and Shah (2002, 2003) studied the mobility of serial manipulators by introducing a virtual ground link between the end-effector and the base so as to form a virtual closed chain. In (Li, & Dai, 2009; Dai, & Shah, 2003), the authors proposed workspace decomposition based on the orientation capability of the manipulator.

Grashof's Theorem has been extended to include more than four-bar chain mechanisms. Grashof's criterion for five bar chain was proposed by Ting (1986). Ting and Liu (1991) extended this work to evaluate the mobility of N-bar chain mechanisms. Nokleby and Podhorodeski (2001)

applied Grashof's criterion for the optimized synthesis for five-bar mechanisms.

In this work we present a simple algorithm for the optimal design of a three-link planar manipulator, using Grashof's criterion. We begin by adding a virtual link to the three-link planar manipulator in order to make it a closed four-bar chain mechanism, so that Grashof's criterion can be applied. We evaluate the generated manipulator designs using dexterity index as a performance measure. Our proposed optimization algorithm generates the required link lengths such that the manipulator has maximum dexterity in the region specified by the user. This region of interest can either be a set of task points or a trajectory. Furthermore, we have also demonstrated, with the help of simulations, the influence of the serial chain link ordering on the dexterous workspace. Finally, we simulate our design under practical conditions such as joint angle limitations.

Nomenclature

- **D:** Dexterity index of the manipulator at a point.
- **D_{Mean}:** Mean dexterity index over a region or trajectory.
- **N:** Number of points along the trajectory.
- **d_x, d_y, d_z:** Dexterity indices about the X, Y and Z axis.
- **α, β, γ:** Yaw, pitch and roll angels of the end-effector.
- **a, b, c, d:** Link lengths of the four-bar kinematic chain.
- **l_1, l_2, l_3:** Link lengths of the three-link planar manipulator.
- **$\theta_1, \theta_2, \theta_3$:** Joint angles of the three-link planar manipulator.
- **d:** Distance between a task-point and base of the manipulator
- **d_{min}:** Minimum distance between a task-point and base of the manipulator.
- **d_{max}:** Maximum distance between a task-point and base of the manipulator.

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